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ADAPTIVE INCREMENTAL HIGH-DEGREE CUBATURE KALMAN FILTER UNDER POOR OBSERVATION CONDITION

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ABSTRACT

This paper is concerned the state estimation problem for nonlinear systems with uncertain process noise covariance and poor observation condition. Firstly, incremental measurement equation is reconstructed by incremental modeling technology. Then, we estimate the uncertain process noise covariance by maximum a posterior (MAP) estimator. Finally, combining with high-degree cubature Kalman filter (HCKF), an adaptive high-degree cubature incremental Kalman filter (AHCIF) is proposed under poor observation condition. The simulations show that the developed algorithm can effectively eliminate the unknown system error. Furthermore, it can also improve estimation accuracy and have a great prospect in the application.

KEYWORDS: adaptive filtering; high-degree cubature Kalman filter; strong tracking filter.

1. INTRODUCTION

Due to the complexity and time variation of the practical application system, state estimation in nonlinear systems is more and more concerned. In order to precisely control the modern systems, nonlinear filtering and estimation have been mightily studied. And a number of algorithms have been developed to solve the nonlinear filtering problem such as extended Kalman filter (EKF) [1,2], unscented Kalman filter (UKF) [3], and cubature Kalman filter (CKF) [4].

EKF is the classical method for nonlinear estimation. It based on linearization theory simply linearizes all nonlinear models so that the traditional linear Kalman filter can be used. But it alse has many disadvantages, such as low accuracy, poor stability, easily divergence, and complexity computing of Jacobian matrix. UKF uses the nonlinear model instead of linearization. It linearizes the random variable and Gaussian distribution while the nonlinear model equations are directly used in the calculations. Compared with the EKF, UKF has the features of lower computation, higher precision and better real-time capability. As in the filtering process, CKS's weight is always positive to ensure the positive definite of covariance. It is suitable to solve the nonlinear filtering problem from low dimension to high dimension and has a wider range of application. Recently, the high-degree CKF (HCKF) is proposed to improve the performance of the CKF [5]. The accuracy and stability performances of HCKF are close to Gauss-Hermite quadrature filter but at lower computational cost. HCKF can achieve higher accuracy than EKF, UKF, and CKF.

Unfortunately, above algorithms must be based on an accurate system model, otherwise it will cause large estimation error and even filter divergence. In practical applications, due to the effect of environment factors, the instability of measurement devices and improper models and parameters, there usually are unknown system errors of the measurement equation and uncertain process noise covariance. Maximum a posteriori (MAP) estimator is an effective noise estimation method. On this basis, a series of adaptive filters is proposed [6,7]. Meanwhile, Fu et al. [8] proposed an incremental system modeling method, which can effectively eliminate the unknown errors of the measurement system. Combining with different basic filters, they developed a series of incremental filters, which have good engineering application value [9-11].

In view of this, this paper studies the nonlinear filter with unknown process noise statistics under poor observation condition. Based on incremental system models, an adaptive high-degree cubature incremental Kalman filter

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(AHCIF) is presented combining MAP estimator. Finally, simulation results show the effectiveness of the proposed methods.

2. PROBLEM FORMULATION

Consider the filtering problem of a nonlinear dynamic system with poor observation condition, which state-space model is described as:

$$\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k)) + \boldsymbol{w}(k) \tag{1}$$

$$\boldsymbol{y}(k) = \boldsymbol{h}(\boldsymbol{x}(k)) + \boldsymbol{\eta}(k) + \boldsymbol{v}(k) \tag{2}$$

where $\boldsymbol{x}(k) \in \Re^n$ is the target state, $\boldsymbol{y}(k) \in \Re^m$ is the measurement value; $\boldsymbol{f}: \Re^n \to \Re^n$ is the nonlinear state evolution process, $\boldsymbol{h}: \Re^n \to \Re^m$ is the corresponding nonlinear measurement mapping; $\boldsymbol{\eta}(k) \in \Re^m$ represents unknown system errors due to measurement equation modeling; process noise $\boldsymbol{w}(k) \in \Re^n$ and measurement noise $\boldsymbol{v}(k) \in \Re^m$ are independent Gaussian noise with zero means, their variances are respective $\boldsymbol{Q}(k)$ and $\boldsymbol{R}(k)$. $\boldsymbol{x}(0)$ is the initial target state with mean $\boldsymbol{x}_0$ and variance $\boldsymbol{P}_0$, and independent of $\boldsymbol{w}(k)$ and $\boldsymbol{v}(k)$.

In order to eliminate unknown system errors $\boldsymbol{\eta}(k)$, we construct the incremental measurement equation as following [8-11]:

$$\begin{aligned} \Delta \boldsymbol{y}(k) &= \boldsymbol{y}(k) - \boldsymbol{y}(k-1) \\ &= \boldsymbol{h}(\boldsymbol{x}(k)) - \boldsymbol{h}(\boldsymbol{x}(k-1)) + \boldsymbol{\eta}(k) - \boldsymbol{\eta}(k-1) + \boldsymbol{v}(k) - \boldsymbol{v}(k-1) \end{aligned} \tag{3}$$

When the sampling rate is high enough, then $[\boldsymbol{\eta}(k) - \boldsymbol{\eta}(k-1)] \to 0$. In this case, we denote $\boldsymbol{z}(k) = \Delta \boldsymbol{y}(k)$ and $\boldsymbol{u}(k) = \boldsymbol{v}(k) - \boldsymbol{v}(k-1)$, then incremental measurement equation can be rewritten as

$$\boldsymbol{z}(k) = \boldsymbol{h}(\boldsymbol{x}(k)) - \boldsymbol{h}(\boldsymbol{x}(k-1)) + \boldsymbol{u}(k) \tag{4}$$

The covariance measurement noise $\boldsymbol{u}(k)$ is $\boldsymbol{R}(k) + \boldsymbol{R}(k-1)$. Obviously, the incremental system composed by equations (1) and (4) can eliminate the unknown system errors effectively.

3. ADAPTIVE HIGH-DEGREE CUBATURE INCREMENTAL FILTER

3.1 Noise covariance estimator

In practice, it is necessary to estimate the noise covariance $\boldsymbol{Q}(k)$. Denote $\hat{\boldsymbol{Q}}(k)$ is an estimation of $\boldsymbol{Q}(k)$, which can be obtained by maximum a posterior (MAP) estimator [6,7]

When $\boldsymbol{Q}(k)$ is constant, its suboptimal MAP estimation $\hat{\boldsymbol{Q}}(k)$ can be recursively calculated by [6]

$$\hat{\boldsymbol{Q}}(k) = \frac{1}{k}\{(k-1)\hat{\boldsymbol{Q}}(k-1) + \boldsymbol{K}(k)\tilde{\boldsymbol{z}}(k)\tilde{\boldsymbol{z}}^T(k)\boldsymbol{K}^T(k) + \boldsymbol{P}(k|k) - \boldsymbol{F}(k)\boldsymbol{P}(k-1|k-1)\boldsymbol{P}(k-1|k-1)\boldsymbol{F}^T(k) \tag{5}$$

where, $\boldsymbol{K}(k)$ is filter gain, $\tilde{\boldsymbol{z}}(k) = \boldsymbol{z}(k) - \hat{\boldsymbol{z}}(k|k-1)$ is measurement innovation, $\boldsymbol{F}(k)$ is the Jacobian matrix of $\boldsymbol{f}(\boldsymbol{x}(k))$, initial value $\hat{\boldsymbol{Q}}(0) = \boldsymbol{Q}_0$.

When $\boldsymbol{Q}(k)$ is time-varying, its suboptimal MAP estimation $\hat{\boldsymbol{Q}}(k)$ can be recursively calculated by [7]

$$\hat{\boldsymbol{Q}}(k) = [1 - d(k-1)]\hat{\boldsymbol{Q}}(k-1) + d(k-1)\left[\boldsymbol{K}(k)\tilde{\boldsymbol{z}}(k)\tilde{\boldsymbol{z}}^T(k)\boldsymbol{K}^T(k) + \boldsymbol{P}(k|k) - \boldsymbol{F}(k)\boldsymbol{P}(k-1|k-1)\boldsymbol{P}(k-1|k-1)\boldsymbol{F}^T(k)\right] \tag{6}$$

where $d(k) = (1-b)/(1-b^{k+1})$, b is the forgetting factor, which satisfies $0.95 < b < 0.99$。

3.2 Adaptive high-degree cubature incremental filter (AHCIF)

In this subsection, we introduce MAP estimator into HCKF (fifth-degree CKF) to estimate and modify process noise statistics, which can reduce the model error and improve the estimation accuracy. Based on nonlinear discrete incremental system by equation (1) and (4), an adaptive high-degree cubature incremental filter is proposed. Suppose that the state estimate $\hat{\boldsymbol{x}}(k-1|k-1)$ and its corresponding covariance $\boldsymbol{P}(k-1|k-1)$ are available at time $(k-1)$, $\hat{\boldsymbol{x}}(k|k)$ and corresponding covariance matrix $\boldsymbol{P}(k|k)$ can be obtained as follows:

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Algorithm: AHCIF

Step1: Initialization

$$\hat{\boldsymbol{x}}(0|0)=\boldsymbol{x}_0,\ \boldsymbol{P}(0|0)=\boldsymbol{P}_0,\ \hat{\boldsymbol{Q}}(0)=\boldsymbol{Q}_0$$

Step2: Time update

1) Calculate the high-degree cubature points $(i=0,1,\cdots,2n^2)$

$$\boldsymbol{x}_i(k-1|k-1)=\boldsymbol{S}(k-1|k-1)\xi_i+\hat{\boldsymbol{x}}(k-1|k-1) \tag{7}$$

where $\boldsymbol{S}(k-1|k-1)$ is the square root factor of $\boldsymbol{P}(k-1|k-1)$. Point set $\{\xi_i\}$ is given by

$$\xi_i=\begin{cases}[0,\cdots,0]^T, & i=0\\ \sqrt{n+2}\cdot\boldsymbol{\varepsilon}_i^+, & i=1,\cdots,\dfrac{n(n-1)}{2}\\ -\sqrt{n+2}\cdot\boldsymbol{\varepsilon}_{i-n(n-1)/2}^+, & i=\dfrac{n(n-1)}{2}+1,\cdots,n(n-1)\\ \sqrt{n+2}\cdot\boldsymbol{\varepsilon}_{i-n(n-1)}^-, & i=n(n-1)+1,\cdots,\dfrac{3n(n-1)}{2}\\ -\sqrt{n+2}\cdot\boldsymbol{\varepsilon}_{i-3n(n-1)/2}^-, & i=\dfrac{3n(n-1)}{2}+1,\cdots,2n(n-1)\\ \sqrt{n+2}\cdot\boldsymbol{e}_{i-2n(n-1)}, & i=2n(n-1)+1,\cdots,n(2n-1)\\ -\sqrt{n+2}\cdot\boldsymbol{e}_{i-n(2n-1)}, & i=n(2n-1)+1,\cdots,2n^2\end{cases}$$

where $\boldsymbol{e}_i$ is the unit vector in $\Re^n$ with the ith element being. Point sets $\{\boldsymbol{\varepsilon}_j^+\}$ and $\{\boldsymbol{\varepsilon}_j^-\}$ are determined by

$$\begin{cases}\boldsymbol{\varepsilon}_j^+\overset{\Delta}{=}\sqrt{\dfrac{1}{2}}\cdot(\boldsymbol{e}_k+\boldsymbol{e}_l):k<l, & k,l=1,\cdots,n\\ \boldsymbol{\varepsilon}_j^-\overset{\Delta}{=}\sqrt{\dfrac{1}{2}}\cdot(\boldsymbol{e}_k-\boldsymbol{e}_l):k<l, & k,l=1,\cdots,n\end{cases}$$

2) Compute the propagated cubature points

$$\boldsymbol{x}_i^*(k|k-1)=\boldsymbol{f}\left(\boldsymbol{x}_i(k-1|k-1)\right) \tag{8}$$

3) Estimate the predicted state and its error covariance

$$\hat{\boldsymbol{x}}(k|k-1)=\sum_{i=0}^{2n^2}\omega_i\boldsymbol{x}_i^*(k|k-1) \tag{9}$$

$$\boldsymbol{P}(k|k-1)=\sum_{i=0}^{2n^2}\omega_i\left[\boldsymbol{x}_i^*(k|k-1)-\hat{\boldsymbol{x}}(k|k-1)\right]\left[\boldsymbol{x}_i^*(k|k-1)-\hat{\boldsymbol{x}}(k|k-1)\right]^T+\hat{\boldsymbol{Q}}(k-1) \tag{10}$$

where the weights ω_i are obtained by

$$\omega_i=\begin{cases}\dfrac{2}{n+2}, & i=0\\ \dfrac{1}{(n+2)^2}, & i=1,\cdots,2n(n-1)\\ \dfrac{4-n}{2(n+2)^2}, & i=2n(n-1)+1,\cdots,2n^2\end{cases}$$

Step3: Measurement update

1) Evaluate the high-degree cubature points

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$$\boldsymbol{x}_i(k|k-1)=\boldsymbol{S}(k|k-1)\xi_i+\hat{\boldsymbol{x}}(k|k-1) \tag{11}$$

where $\boldsymbol{S}(k|k-1)$ is the square root factor of $\boldsymbol{P}(k|k-1)$.

2) Compute the propagated cubature points

$$\boldsymbol{z}_i(k|k-1)=\boldsymbol{h}(\boldsymbol{x}_i(k|k-1))-\boldsymbol{h}(\hat{\boldsymbol{x}}(k-1|k-1)) \tag{12}$$

3) Calculate the predicted measurement

$$\hat{\boldsymbol{z}}(k|k-1)=\sum_{i=0}^{2n^2}\omega_i\boldsymbol{z}_i(k|k-1) \tag{13}$$

4) Compute the innovation covariance matrix

$$\boldsymbol{P}_{zz}(k|k)=\sum_{i=0}^{2n^2}\omega_i\left[\boldsymbol{z}_i(k|k-1)-\hat{\boldsymbol{z}}(k|k-1)\right]\left[\boldsymbol{z}_i(k|k-1)-\hat{\boldsymbol{z}}(k|k-1)\right]^T+\boldsymbol{R}(k)+\boldsymbol{R}(k-1) \tag{14}$$

5) Evaluate the cross-covariance matrix

$$\boldsymbol{P}_{xz}(k|k)=\sum_{i=0}^{2n^2}\omega_i\left[\boldsymbol{x}_i(k|k-1)-\hat{\boldsymbol{x}}(k|k-1)\right]\left[\boldsymbol{z}_i(k|k-1)-\hat{\boldsymbol{z}}(k|k-1)\right]^T \tag{15}$$

6) Calculate the Kalman gain

$$\boldsymbol{K}(k)=\boldsymbol{P}_{xz}(k|k)\boldsymbol{P}_{zz}^{-1}(k|k) \tag{16}$$

7) Estimate the updated state and its error covariance

$$\begin{cases}\hat{\boldsymbol{x}}(k|k)=\hat{\boldsymbol{x}}(k|k-1)+\boldsymbol{K}(k)[\boldsymbol{z}(k)-\hat{\boldsymbol{z}}(k|k-1)]\\ \boldsymbol{P}(k|k)=\boldsymbol{P}(k|k-1)-\boldsymbol{K}(k)\boldsymbol{P}_{zz}^{-1}(k|k)\boldsymbol{K}^T(k)\end{cases} \tag{17}$$

Step4: Noise covariance estimation

Estimate $\hat{\boldsymbol{Q}}(k)$ according to equation (5) or (6).

4. SIMULATION EXAMPLE

In this section, a simple simulation example is used to verify the effectiveness of the proposed AHCIF compared with adaptive unscented incremental filter (AUIF) in [10]. Consider the following univariate nonstationary growth model (UNGM)[10,11]

$$x(k)=0.5x(k-1)+\frac{2.5x(k-1)}{1+x^2(k-1)}+8\cos(1.2k)+w(k) \tag{18}$$

$$y(k)=\frac{x^2(k)}{20}+\eta(k)+v(k) \tag{19}$$

Where $w(k)$ and $v(k)$ are independent Gaussian white noises. $\eta(k)$ is the unknown system error of measurement equation. In our simulation, $R(k)=1$, $\eta(k)=4$, $b=0.95$, initial state $x(0|0)=0$ and its covariance $P(0|0)=10$, simulation step is 100. Process noise covariance $Q(k)$ is time varying, which is satisfied

$$Q(k)=\begin{cases}1, & 1\le k\le 50\\ 2, & 50<k\le 100\end{cases}$$

The simulation results are illustrated by Figure 1 and Figure 2. Synchronously, the mean absolute error of two methods is given by Table 1.

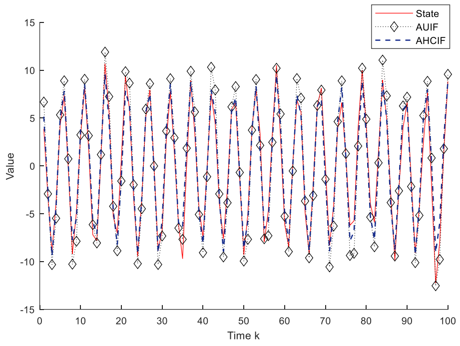

Figure 1 Estimation curves of two algorithms

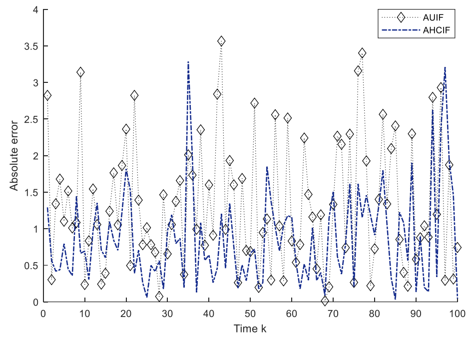

Figure 2 Absolute error curves of two algorithms

Table 1. Mean absolute error of two algorithms

Algorithm	AUIF	AHCIF
Mean absolute error	1.3200	0.8720

From Figure 1 and Figure 2, it is easy to see that both methods can effectively estimate the state to some extent, because both of them can eliminate the unknown system errors $\eta(k)$ and estimate the covariance of process noise. Meanwhile, AHCIF has better filtering effect than AUIF. The mean absolute error in Table 1 shows that the mean absolute error of AUIF is 1.3200, while that of AHCIF is 0.8720, which improves the accuracy by 43.72%. This also shows that the method proposed in this paper is more effective.

5. CONCLUSION

This article investigates the problem of nonlinear estimation with unknown variance of process noise under poor observation condition. In order to deal with these uncertainties, incremental modeling technology and MAP estimator are adopted in high-degree cubature Klamn filter (HCKF), a novel adaptive HCKF is developed, which is called AHCIF. The experimental results demonstrate the effectiveness of the proposed algorithms. It will be an interesting future research topic to consider the fusion algorithm for multi-sensor systems.

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